Focusing of light beyond the diffraction limit by randomly distributed graded index photonic medium

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Sub-wavelength focusing of light holds great potential in various applications of science and engineering, including nanolithography, optical microscopy, optical measurements, and data storage. In the present paper, we propose a new concept to obtain sub-wavelength focusing of light by using structures composed of all-dielectric materials. The approach utilizes the design of an inhomogeneous refractive index profile with random distributions of individual elements occupying the unit cells of two-dimensional photonic crystals (PCs). Light focusing phenomenon is both systematically and quantitatively analyzed at different selected frequencies and we show that the randomly generated graded index (GRIN)-like photonic medium provides light focusing in air with a spot size below $\lambda/3$, where $\lambda$ is the wavelength of light. The numerically obtained minimum spot size is equal to 0.260$\lambda$. Gaussian probability function is used to implement numerous random designs to investigate the optical characteristics of the photonic medium. Specific attention is paid to the sub-wavelength focusing properties of the designed random PC structures. The random ingredient of GRIN PC provides not only narrow focusing of light but also off-centered or asymmetric light focusing at the back side of the structure. Experimental verification conducted at the microwave region shows good agreement with the numerical results. Published by AIP Publishing.

I. INTRODUCTION

The pioneering works in Refs. 1 and 2 paved the way for a new branch of light-wave science that allows manipulating photons with a new type of dielectric structures called photonic crystals (PCs). The spatial modulating of the refractive index results in the appearance of energy bands and it enables control of both the temporal and spatial dispersion properties of light waves, which in turn provides different functionalities that can tailor the propagation of light. The PC concept has been extensively studied in the photonics field since 1987. Light interaction with the PC structures creates unique properties that cannot be observed in a standard optical medium. For instance, for particular designs, PC structures may provide slow-light effect, optical confinement with high Q-factor, super-prism effects, self-collimation phenomena, and sensitive bio-chemical sensing abilities.3

In addition to PCs, aperiodically ordered or disordered photonic structures have attracted great attention because of their intriguing optical characteristics.4–6 Aperiodic and disordered structures compared to periodic structures can provide additional flexibilities and properties for engineering the optical response of the designed devices. The study of light propagation in random media has become a very popular area of research in recent decades.7,8 Light scattering in disordered/random media may provide strong photon localization known as Anderson localization. Such structures have potential in optical systems, imaging, random lasing, and solar energy applications.9–13 Moreover, thanks to a recent study, a high resolution spectroscopy is designed by adding an intentionally controllable disorder to a structure.14 In addition, random lasers with disordered media and high-Q cavity with random localized disorder have been introduced recently to the literature.15,16

Since the studies on disordered and random photonic structures are continued, there is interest in aperiodic and quasi-periodic structures as well. The discovery of quasicrystals in condensed matter by Shechtman in 1984 established the foundation of the new field of aperiodically ordered crystals called “quasicrystals” and initiated a new research field in photonics.17 The newly created photonic quasicrystals (PQCs) field has received an increasing amount of attention in recent years.18 The massive research being conducted in this area has been motivated by the idea of generating photonic bandgaps at low refractive index contrast that allows using a great variety of materials in photonics, including biomaterials.19 On the other hand, the theory of quasiperiodicity of PCs can be considered as a competition between two spatial structural characteristics: self-similarity and aperiodicity where the first is responsible for a long range field pattern extension and the latter can be attributed to localized states.20 With the help of such PC designs, unique optical properties appear in the case of the transmission, reflection, refraction, localization, and radiation of photons as well as the symmetry in the Fourier space, nonlinear optical, and diffraction characteristics.18 In this sense, the frequency domain characteristics of a different type of PQCs with eightfold,
tenfold, and twelvefold symmetries in two dimensions have been investigated.\textsuperscript{21–23}

A conventional way of focusing light requires the usage of either mirrors or lenses with curved surfaces and constant refractive indices.\textsuperscript{24} Reflection and refraction are responsible for bringing light into focus in these cases. Although conventional lenses are able to provide large degree of freedom of light manipulation, their resolution is inherently limited by the diffraction phenomenon, i.e., the wave nature of light has a limitation to focus light below a certain spot size that cannot be smaller than half of the operational wavelength. To overcome the diffraction limit, i.e., to achieve sub-wavelength focusing or imaging, artificial structures such as super-lenses can be constructed. So far, various notable approaches based on negative refraction,\textsuperscript{25,26} Fabry-Perot resonances,\textsuperscript{27} time reversal in plasmonic nanostructures,\textsuperscript{28} superlenses,\textsuperscript{29–33} hyperlenses,\textsuperscript{34–37} a metal sandwiched tapered dielectric structure,\textsuperscript{38} super oscillation,\textsuperscript{39} aperiodic metallic waveguide array,\textsuperscript{40} nanoslit structures,\textsuperscript{41} metamaterials,\textsuperscript{42} and photonic nano-jets\textsuperscript{43} have been proposed to achieve focusing beyond the diffraction limit, i.e., sub-wavelength focusing.

The introduction of the graded index concept (GRIN) has further enriched the ability of efficient light manipulation in optical applications.\textsuperscript{44–47} The main supremacy of GRIN optical focusing elements over conventional lenses is that the GRIN lenses can exhibit a strong light focusing effect with a flat front/back surface.\textsuperscript{48,49} The majority of the studies concentrated on using GRIN PC, where the index modulation satisfies a specific function or profile that is obtained by the ordered distribution of PC rods. In this paper, we consider a GRIN like PC medium with a refractive index profile that obtained by a probability distribution function instead of a specific refractive index profile. It should be noted that the preliminary data of the current work was presented in an international conference.\textsuperscript{50} The refractive index modulation is achieved by introducing random disordering according to the Gaussian probability distribution function to the location of PCs unit cells on the transverse direction. We demonstrate that the random modulation of PC super cells can be considered as an additional enhancement on the light manipulation in terms of output beam compression. As a result, sub-wavelength light focusing with a focal spot size equal to 0.26\% is achieved, which is considerably smaller than the spot sizes obtained in previously studied GRIN PC lens structures.\textsuperscript{51–53} A normalized bandwidth (the ratio of the frequency bandwidth to the center operational frequency) as high as 26.1\% is also verified, showing that the proposed structure can also outperform non-GRIN PC lens structures based on negative refraction, in terms of operational bandwidth (e.g., 4.3\%, 0.7\%, and 3.5\% for Refs.\textsuperscript{54,55, and 56}; respectively). Moreover, the usage of all-dielectric materials in the present study enables absorption-free low loss operation, compared to metamaterial and plasmonic based sub-wavelength focusing structures. An experimental study performed at the microwave region validates the numerical findings. We expect that the proposed structure will have various important applications in areas, where highly focused light beams together with broadband and low loss operation are desired. Based on the statistical analysis (which will be presented in Sec. III), we further expect that the present study will reduce the fabrication difficulties associated with strict structural parameters, due to the lack of any requirement for obeying a definite rod distribution function. Finally, one can use the scalability of the structure to target different operating bandwidths within the electromagnetic spectrum. Similar results can be obtained by using phononic crystals that deal with acoustic waves.

II. LIGHT FOCUSING: RANDOMLY DISTRIBUTED GRADED INDEX PC MEDIUM

Among various light manipulation scenarios, one of the widely targeted problems is to improve focusing characteristics of photonic structures. However, it is difficult to control light if the structure has wavelength-scale geometrical features along with a high-contrast refractive index variation instead of uniform/homogeneous medium, where conventional optical components fail due to the limitation caused by the diffraction nature of the light, i.e., limited to \( \lambda/(2 \text{NA}) \), where \( \lambda \) is the wavelength of light and NA is a numerical aperture of the focusing lens.

Recent improvements in the light focusing field showed that the propagation of light can be efficiently controlled by the GRIN optics concept. The non-uniform index distribution enables light rays to follow curved trajectories in an inhomogeneous GRIN medium. Therefore, curving the light path paves the way for the optical characteristic such as focusing/collimating and diverging/spreading such that the same manipulations can be achieved with the conventional optical elements possessing curved interfaces.

In order to explain the behavior of light waves with and without the GRIN medium, Ray theory can be considered as a possible analysis method. A rapid insight on the modeling of light propagation through a conventional GRIN medium (where the refractive index is locally homogeneous and index profile of the medium is usually in a quadratic shape) can be obtained with Ray optics by solving Eikonal equation for any type of index distribution.\textsuperscript{46} Detailed explanation on solving the Eikonal Equation can be found in our previous works.\textsuperscript{37,49} Propagating light rays oscillate in a sinusoidal manner along the optical axis (OA) in a conventional GRIN medium. Therefore, a critical parameter such as the oscillation period can be calculated as \( P = 2 \pi / \alpha \) where \( \alpha \) is the gradient parameter of the GRIN medium. To have a thin GRIN medium to focus the incident light, one needs to have a length that is smaller than the quarter of the period of oscillation \( P/4 \).\textsuperscript{56,62} It is necessary to move from a continuous inhomogeneous medium to a discrete version to implement index gradient with two types of media (air and dielectric) to mimic continuously varying inhomogeneous index profile. Artificially designed periodic structures, i.e., PCs make it possible if we play with the filling factor of the structure.\textsuperscript{47–50} To perform the mimicking process exploiting of PC dielectric rods, dispersion engineering or Maxwell Garnet effective medium theory can be used.\textsuperscript{51}
intentionally disturbed by locating the dielectric rods position in a random manner. It is expected that the idea of merging random disorder with index gradient concept can yield rich light manipulation capabilities and not only on-centered but off-centered focusing but also directional beaming.

In this study, probability density function with Gaussian distribution is utilized to design the proposed randomly distributed PC (RDPC) structure. The locations of RDPC dielectric rods’ are distributed pseudo-randomly in the transverse y-direction. The distance between adjacent columns/layers along the propagation x-direction is fixed to a, where a is the lattice constant. In such a way, one can deduce the effective refractive index of the designed structure that is modulated only in the transverse y-direction. In other words, the gradient of the dielectric filling ratio in single column/layer along the y-direction varies according to a $G(y)$ Gaussian distribution probability. Figure 1(a) represents the Gaussian probability function $G(y)$ graph that stipulates the positioning of the RDPC rods and Fig. 1(b) shows the corresponding schematic view of the proposed RDPC columns/layers in which rods’ positions obeys to the Gaussian probability distribution function. The function $G(y)$ can be formulated as follows:

$$G(y) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(y - \mu)^2}{2\sigma^2}\right),$$

where $\sigma$ is the standard deviation and $\mu$ is the mean value of the distribution. In order to ensure a relatively high effective index near the optical axis, the mean value was set equal to $\mu = 0$, whereas the standard deviation was chosen so that the resulting index distribution would mimic a GRIN-like medium with a smoothly decreasing index towards the edges. However, we should note that, since an overlap constraint must be used (to avoid rod overlaps), the width of the resulting index distribution will be larger than that of the targeted distribution, as can be observed when comparing Figs. 1(a) and 1(c). The discrepancy is a direct consequence of the fact that the rod dimensions are comparable with the structural height, as opposed to the analytical assumption where the individual elements are expected to be infinitely small. In this regard, a sequential optimization has been performed and a standard deviation equal to $\sigma = 3a$ was determined to fulfill the above mentioned criteria. The corresponding height (h) and width (w) of the structure are equal to $L_y = 30a$ and $L_x = 10a$, respectively. To avoid complexities in the practical implementation of RDPC rods, the refractive indices are kept the same and equal to $n = 3.13$ and radii of the dielectric rods are fixed to $r = 0.20a$ (the RDPC rods were considered alumina rods for the microwave experiment).

To extract the effective index distribution of the RDPC structure, the well-known effective medium theory is used. By knowing the exact positions of the RDPC rods in a single layer, one can calculate the corresponding effective refractive index on that location. Effective index profiles are extracted and plotted in Fig. 1(c) for the given randomly distributed different columns/layers of the RDPC shown in Fig. 1(b). As can be seen from Figs. 1(b) and 1(c), the density and effective refractive index of the structure tends to be higher while approaching optical axis (OA). Therefore, the distribution of the RDPCs along the transverse direction reveals an inhomogeneous PC medium whose effective index profile resembles a non-uniform parabolic shape. We should note that the deduced effective medium will be valid at the long-wavelength regime (at which our next analyses will be performed). Due to the isotropy of such a medium, the deduced effective index will be valid even for dielectric rods that are separated by relatively large distances, as was previously studied, for example, in Ref. 49.

The proposed RDPC structures are numerically implemented to observe the transmission response by the use of finite-difference time-domain (FDTD) method. By assuming that every RDPC structure has different time and frequency responses for propagating beams, we have calculated the power transmittance for five different RDPC structures; thus, it enables us to observe the dependency between structural width and overall transmittance. The increment of the width of the RDPC structure is schematically represented in Figure 2 as an inset where the numbers at the top of each part designated the length of the structure. To calculate the power transmission efficiency of the structure, a broad band pulse with a Gaussian shape with TM polarization mode is launched. A detector is located at the end of the structure to numerically obtain the transmission spectra.

FIG. 1. (a) Plot of the Gaussian probability density function for random distribution of the rods’ locations. (b) The schematic view of the proposed RDPC structure. Red arrow depicted at the bottom of the structure shows the propagation direction of the source. (c) Random approximation of effective refractive index along the transverse y-direction for every randomly disordered column line.
The transmission efficiencies are calculated and normalized by taking the ratio of detected and incident power. Calculated transmission plots are depicted in Fig. 2. The careful inspection of the transmission spectra for five different RDPC structures reveals that at low frequencies below $a/\lambda = 0.20$, there is a high transmission region which is followed by a directional bandgap. As can be seen from the transmission plot in Fig. 2, the directional bandgap strongly appears for the structure with width value equal or larger than 4.0$a$. The second and third high transmission intervals are observed at frequency intervals $(0.416 \leq a/\lambda \leq 0.562)$ and $(0.681 \leq a/\lambda \leq 0.784)$, respectively. The appearance of the multiple gaps is due to the intact periodicity along the propagation direction.

The electromagnetic field propagation and phase front transformations inside the generated RDPC are examined next. The structure is tested under the long wavelength region. The longitudinal length of the structure was intentionally taken to be long enough, $L_x = 100a$, to clearly observe the evaluation characteristic of the propagating wave within the RDPC structure. The extracted steady state fields of propagating beams are given for different frequencies in Figs. 3(a)–3(d). As can be seen, the planar phase fronts at the input of the structure transforms to the convex like shape (converging effect emerges) as it propagates. Due to the random distribution of the index profile, the light guiding behavior demonstrates a more complex pattern compared to the conventional GRIN PC. Hence, at each propagation distance $a$, light encounters a slightly different index form and, finally, well predicted and oscillatory light propagation does not occur. Even under this complex light propagation mechanism, we can approximately determine the length of the structure necessary for focusing effect. The calculated quarter period values for the frequencies of $\{0.10 \leq a/\lambda \leq 0.13\}$ vary between $P/4 = 28a$ and $P/4 = 29a$. The wave propagates until the quarter period distance with a converging phase front and starts to diverge from this point on. Therefore, by proper termination of the structure, one can get converging or diverging wave fronts at the output of the structure. Special GRIN termination cases are investigated in detail in Ref. 47.

### III. STUDY OF RANDOMLY DISTRIBUTED PC FOCUSING LENS

In this section, the focalization concept exploiting RDPC medium is considered and used to achieve a focusing effect, the structural length is selected to be less than the calculated quarter pitch value as mentioned before. Previously, the oscillation periods are calculated for propagating beams for the operating frequency interval of $\{0.10 \leq a/\lambda \leq 0.13\}$ and corresponding quarter periods are obtained as $P/4 = 28a – 29a$. As can be seen in Fig. 3, the propagating beam phase fronts continue to converge until the length of the structure reaches $28a – 29a$ values for different frequencies. Therefore, as was described in Section II, to ensure the converging and focusing behavior of the incident rays at the back plane of the structure, we adjusted the length of the RDPC structures to be equal to $10a$.

To observe the steady state response of the proposed structure, a TM polarized continuous-wave source, placed with a distance of 0.5$a$ before the input surface, was used to excite the proposed RDPC with fixed structural parameters $L_y = 30a$ and $L_x = 10a$. Figure 4 provides a collection of spatial intensity profiles and cross sectional plots for operating frequencies of $a/\lambda = 0.11$ and $a/\lambda = 0.13$. Specifically, for the normalized operational frequency $a/\lambda = 0.11$, Figs. 4(a) and 4(b) show the spatial intensity distribution and the cross sectional intensity profile of the focal spot with $0.361\lambda$ FWHM value along the transverse $y$-direction, respectively. Figure 4(c) shows the intensity variation along the optical axis (longitudinal $x$-direction). As shown in Figs. 4(a) and 4(c), the distance between the focal spot and the rear surface of the

![Fig. 2. Calculated transmission efficiencies for different widths (different numbers of columns) of the RDPC structure. Different column numbers with widths 1.0$a$, 4.0$a$, 7.0$a$, 10$a$, and 13$a$ are shown schematically as an inset.](image)

![Fig. 3. Instantaneous electric field snapshots of the designed RDPC structure with a length of 100$a$, operating at the selected frequencies of (a) $a/\lambda = 0.10$, (b) $a/\lambda = 0.11$, (c) $a/\lambda = 0.12$, and (d) $a/\lambda = 0.13$.](image)
RDPC along the optical axis is defined as the back focal length $D_F$. One can infer from the spatial intensity distribution in Fig. 4(a) that the incident light gets focused at the back plane of the RDPC structure. This behaviour is consistent with the formation of the converging wave fronts, induced by the termination of the oscillatory propagation, as was discussed in Section II. Similarly, Figs. 4(d)–4(f) provide information about the focusing behavior of the designed RDPC structure operating at frequency of $a/k = 0.13$. As can be seen from Figs. 4(e) and 4(f) corresponding FWHM (0.328$\lambda$ at operating frequency of $a/k = 0.13$) and $D_F$ values are decreased. From Figs. 4(a) and 4(d), one can see that while increasing the operating frequency the modulation of propagating field takes more complex shape due to multiple enhanced light scattering events, i.e., the smaller wavelength senses strongly the structural disorder. Furthermore, while the frequency is increased from $a/k = 0.10$ up to $a/k = 0.13$, the focal point moves closer to the back surface of the structure and thus, corresponding back focal length decreases. The changes in FWHM values imply that while the focal point moves towards the end facet of the RDPC structure and multiple side lobes are starting to appear, the focusing capability of the configuration is enhanced and thus spot sizes become smaller. Strong focusing of light at the focal point as compared in Figs. 4(b) and 4(e) is associated with fast decaying of intensity along the longitudinal direction.

On the other hand, each column of super cell has different structural orientation, i.e., the minimum value of the gradient of the effective refractive index of each super cell deviates at around the optical axis. This means that because of the random distribution of the RDPC rods, the density and effective refractive index of the structure tend to be higher while approaching the optical axis. In this case, asymmetric light propagation can appear at the back focal plane and directional or off-centered emissions may exist at the end of the structure. Beam steering and directional light emission can be an output response of the RDPC structure. Figure 5 is generated to show on and off axis focusing (directional emitting and steering of output beam) fluctuations for the RDPC iterations. By iterations, we mean different RDPC structures, which are generated randomly independent of each other. The insets in Fig. 5 represent intensity field distributions nearby the focal points at the specified operating frequencies. These insets are chosen to show the maximum beam steering effect occurring at the corresponding frequencies. Dashed lines within the insets show the optical axes.

In order to statistically evaluate the spot sizes and the longitudinal positions of the focal spots, we further inspected 250 different iterations of RDPCs. Randomly disordered structures are illuminated with a continuous-wave source operating at the frequencies of $a/k = [0.10, 0.11, 0.12, 0.13]$. The calculated corresponding FWHM value deviations are represented using histogram plots in Figs. 6(a)–6(d). As can be seen in Fig. 6(a), almost all the FWHM values are under $0.50\lambda$, which means a randomly generated structure operating at $a/k = 0.10$ acts as a sub-wavelength focusing lens and the minimum value of FWHM is equal to $0.260\lambda$. To quantitatively evaluate the subwavelength focusing ability,
we define sub-wavelength focusing success as the ratio of the number of iterations that have a focal spot with a FWHM value smaller than $a/2$, to the total iteration number. While increasing the frequency to $a/\lambda = 0.11$, the structures show 90.8% sub-wavelength focusing success and the minimum value of FWHM increases to $0.335\lambda$ compared to the $a/\lambda = 0.10$ case. For other frequencies, in Figs. 6(c) and 6(d), $a/\lambda = 0.12$ and $a/\lambda = 0.13$, sub-wavelength focusing success decreases to 88%–89%. According to the focal length data, one can deduce that while increasing the operating frequency the focal point gets more distant with respect to the output back plane. Based on the reported results, we can deduce that merging the idea of randomness and GRIN can achieve sub-wavelength focusing with plenty of different design parameters. The obtained spot sizes indicate that a well-optimized semi-disordered PC medium can focus light down to one fifth of its wavelength,\textsuperscript{59} the fabrication of such precisely placed dielectric rods at the nano- or micro-scale still remains as an open task. On the other hand, the concept we propose may alleviate such fabrication challenges, due to the relaxed requirement of definite rod positions.

Two different physical mechanisms may exist inside such a type of RDPCs, which govern the strong focusing of light: The first one is the provided index gradient due to randomly placed dielectric RDPC rods. Although the positioning of dielectric PCs is random along the transverse y-direction, as shown in Fig. 1(b), the overall effective index distribution of RDPC structure is concentrated around the center so that the density of RDPC rods decreases towards the edges according to the distribution function $G(y)$ given in Eq. (1). The presence of the index gradient effect can also be observed in the semi-periodic field oscillations as shown in Fig. 3. Furthermore, since the center of average index profile

![FIG. 5. Fluctuation of the focal point around optical axes of 100 RDPC structures operating at frequencies of $a/\lambda = [0.10, 0.11, 0.12, 0.13]$. The insets represent intensity field distributions of the steered (off axis focused) output beam.](image)

![FIG. 6. Histogram of the calculated FWHM values at normalized frequencies (a) $a/\lambda = 0.10$, (b) $a/\lambda = 0.11$, (c) $a/\lambda = 0.12$, and (d) $a/\lambda = 0.13$, showing the number of occurrences out of 250 RDPC iterations.](image)
for each RDPC layers may deviate from the exact transverse center (see Fig. 1(c)), the disordered graded index medium may produce a focus with a slight distortion in orientation, i.e., an off-/on- axis focus. The other mechanism that may strengthen the focusing capability is based on additional scattering of long-wavelength photons with randomly located rods. It is worth noting that in the homogenization regime, the impact of a random scatterer positioning is expected to be relatively weak on wave transport. As can be observed from field propagations in Figs. 3 and 4, there is not a continuously varying curved phase fronts inside the RDPC. Light is spatially confined at certain locations that act as trapping points giving rise to hot-spots. Instead of side-lobe free Gaussian beam at the focal point, the focused light is accompanied by side lobes due to interference of beam radiating from the hot-spots located at around the optical axis. In other words, random scattering causes additional interference of light and yields speckle spots that vary in size and intensity around certain average values. Such a multi-beam interference may provide an enhancement in terms of output beam narrowing, and therefore, the corresponding FWHM values may reduce. Similar multiple beam interference mechanism has been also observed in Ref. 59, where subwavelength focusing of light is achieved by applying a sophisticated optimization algorithm to the design of an all-dielectric PC lens structure. As a result, the combination of the above-mentioned physical phenomena, enhancement in terms of focusing as well as steering of output beam can be achieved by using RDPCs.

In summary, the detailed calculation of the focusing dynamic characteristic are collectively presented for all 250 iterations in Table I.

The statistical evaluations of the proposed RDPC structure have shown that the structure preserves its focusing behaviour despite its strong disorder. Although focusing characteristics, such as the spot size and the spatial position of the focal spot, may vary among different iterations, the statistical analysis of large number different structures has revealed that the quantitative values of these characteristics are concentrated around distinct values with specific deviations depending on the operational frequency.

IV. EXPERIMENTAL VERIFICATION

To verify the numerical results, we performed the experimental realization of the designed RDPC structure at the microwave region. The studied structures are composed of cylindrical Alumina rods with a refractive index of $n = 3.13$ and a diameter of $d = 3.175$ mm. The corresponding lattice constant is then set to $a = 7.75$ mm. Figures 7(a) and 7(b) show the schematics of the experimental setup and photographic illustration of the investigated RDPC lens configurations. The corresponding lens configurations are chosen from 250 RDPC iterations in such a way that on axis, up steering and down steering with strong focusing effects could be observed. As can be seen in Fig. 7(b), three different RDPC structures are designed, which allow one obtaining (upper) on axis, (middle) down steer, and (lower) up steer focusing. A vector network analyzer, Agilent E5071C ENA was used to generate a wave source and record the intensity field of the focused wave. In order to excite the structure and measure the steady-state intensity distribution the horn and monopole antennas are employed, respectively. Operating frequency of the antennas ranges between 3.5 GHz and 5.5 GHz. We carefully tried to match the numerical and experimental excitation conditions by placing the source at the front face of the RDPC lens in both cases.

Additionally, microwave absorbers are placed at around the structure to reduce any possible back reflections. A TM polarized wave was launched to the designed RDPC lens; as stated earlier, the intensity distribution was obtained with the monopole antenna, by measuring the focused field intensity at the air back focal plane of the constructed structure. The monopole antenna was placed parallel to the rods (perpendicular to the x-y plane), and the tip of the antenna was placed to be at the half height of the alumina rods. The steady state intensity field at the back focal plane was measured by moving the monopole antenna 14 cm in the y- and 3 cm in the x-directions with spatial steps equal to 2 mm. Figures 7(c) and 7(d) represent the measured electric intensity field and its cross-sectional intensity profiles of the focused beam operating at frequency of 3.87 GHz. The measured FWHM value from the experimental data is 0.269$, and maximum side lobe value stays below the normalized intensity value of 0.40.

In order to show the focused beam steering, we also perform the same experimental steps for the up steering and down steering RDPC lenses for the same frequency of 3.87 GHz (normalized frequency is $a/\lambda = 0.10$). Similarly, Figs. 7(e), 7(f) and 7(g), 7(h) show the experimental results of down and up steering RDPC structures, respectively. In order to show the broadband operation of the designed RDPC lenses as a subwavelength focusing apparatus, we have performed experimental measurements for a wide range of operating frequency interval of [3.87 GHz, 5.03 GHz] ($a/\lambda = [0.10, 0.13]$). The measured results are collectively represented in detail in Table II. Inspecting Table II, for 3 different structures at

### Table I. Numerical results of focusing ability of randomly disordered graded index PC media.

<table>
<thead>
<tr>
<th>Normalized frequency ($a/\lambda$)</th>
<th>0.10</th>
<th>0.11</th>
<th>0.12</th>
<th>0.13</th>
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<tbody>
<tr>
<td>Sub-wavelength focal success rate</td>
<td>250/100%</td>
<td>227/90.8%</td>
<td>215/86.0%</td>
<td>210/84.0%</td>
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<tr>
<td>FWHM ($\lambda$)</td>
<td>Max/min</td>
<td>0.490/0.265</td>
<td>0.545/0.275</td>
<td>0.660/0.294</td>
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<tr>
<td></td>
<td>Mean (N)</td>
<td>0.399</td>
<td>0.445</td>
<td>0.452</td>
</tr>
<tr>
<td></td>
<td>Standard deviation (N)</td>
<td>0.046</td>
<td>0.045</td>
<td>0.053</td>
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<tr>
<td>Back focal length $\Delta F$ (a)</td>
<td>Max/min</td>
<td>2.2/0</td>
<td>2.95/0</td>
<td>3.35/0</td>
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<tr>
<td></td>
<td>Mean (N)</td>
<td>0.936</td>
<td>1.552</td>
<td>1.622</td>
</tr>
<tr>
<td></td>
<td>Standard deviation (N)</td>
<td>0.714</td>
<td>0.634</td>
<td>0.925</td>
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normalized frequency of 3.87 GHz, we observe the strong focusing effect with FWHM values of 0.269λ, 0.323λ, 0.330λ, respectively. When we inspect the FWHM values of the experimental results within the operating frequency of 3.87GHz–5.03 GHz as summarized in Table II, we see the broadband nature of the designed photonic structure as a sub-wavelength k/3 focusing device. Such a broadband behaviour can be attributed to the constant effective indexes of the RDPC rod elements in their broad homogenization regime.47

The microwave experiment results have validated the numerical findings such that the RDPC structure can provide absorption-free and broadband sub-wavelength focusing of electromagnetic waves with directional emitting and steering properties. Even though we have implemented a Gaussian distribution for determining the locations of the PC rods, other functional distributions can also be deployed.

V. CONCLUSIONS

To conclude, sub-diffraction limited focusing ability of an inhomogeneous GRIN-like RDPC medium is studied numerically and experimentally. A controlled random concept was utilized instead of fully random fluctuations of rods locations. The idea of merging GRIN and randomly distribution concept is realized to obtain sub-wavelength focusing and we demonstrated that it is possible to obtain a focusing of light beyond the diffraction limit with a random modulation graded index profile. The proposed structure is designed by 2D PC rods and time domain analyses are conducted using the FDTD method. Investigated configurations provide a strong sub-wavelength focusing effect and wavelength dependence of the focusing effect are numerically reported and verified experimentally at the microwave region. A numerically obtained minimum FWHM value is equal to 0.260λ and experimentally measured one is equal to 0.269λ. In addition, a sub-wavelength focusing success rate of more than 90% within the wide frequency interval a/λ = [0.10–0.13] for simulated cases was achieved. Moreover, the broadband sub-wavelength focusing nature is also demonstrated experimentally where the measured FWHM stays under k/3 value for an

<table>
<thead>
<tr>
<th>Op. frequency</th>
<th>FWHM values</th>
</tr>
</thead>
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<tr>
<td>a/λ, GHz</td>
<td>On axis RDPC</td>
</tr>
<tr>
<td>0.10 3.87</td>
<td>0.269λ</td>
</tr>
<tr>
<td>0.11 4.25</td>
<td>0.277λ</td>
</tr>
<tr>
<td>0.12 4.64</td>
<td>0.307λ</td>
</tr>
<tr>
<td>0.13 5.03</td>
<td>0.273λ</td>
</tr>
</tbody>
</table>

FIG. 7. (a) Schematic view of the experimental setup and (b) photographs of the manufactured RDPC lenses (on axis, down steer, and up steer focusing configurations) at the microwave regime. Experimentally measured (c) electric field intensity and (d) transverse cross-sectional profiles at a focal point at the operating frequency of 3.87 GHz. Similarly, (e), (f) and (g), (h) are representation of measured electric field intensities and cross-sectional intensity profiles of down and up steering RDPC structures, respectively. Detailed tabulation of the quantitative results of the realized experiment is given in Table II.
operating frequency range between 3.87 GHz and 5.03 GHz. Asymmetric and directional emission can also be obtained with the designed structures. Optical microscopy imaging, micro-particle tweezing, lithography, and laser machining are some of the research areas or devices that can benefit from the outcome of the research efforts conducted with the randomly oriented index gradient medium. Moreover, the results presented here can be applied to other fields such as acoustic waves, and the application domain can be extended into non-linear applications.

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